

Modelling of MeV alpha particle energy transfer to lower hybrid waves

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Abstract. The interaction between a lower hybrid wave and a fusion alpha particle displaces the alpha particle simultaneously in space and energy. This results in coupled diffusion. Diffusion of alphas down the density gradient could lead to their transferring energy to the wave. This could, in turn, put energy into current drive. An initial analytic study was done by Fisch and Rax. Here we calculate numerical solutions for the alpha energy transfer and study a range of conditions that are favourable for wave amplification from alpha energy. We find that it is possible for fusion alpha particles to transfer a large fraction of their energy to the lower hybrid wave. The numerical calculation shows that the net energy transfer is not sensitive to the value of the diffusion coefficient over a wide range of practical values. An extension of this idea, the use of a lossy boundary to enhance the energy transfer, is investigated. This technique is shown to offer a large potential benefit.

1. Introduction

Lower hybrid current drive has been an attractive alternative to inductive current drive, but for reactors there has been concern that fusion alpha particles would be a strong absorber of lower hybrid power [1, 2]. Recently, however, it has been pointed out [3] that under the right conditions the alpha population can actually amplify the wave, thereby harnessing some of the energy of the alpha population for the current drive. The necessary conditions require that there be a density gradient of fast alphas, and that k_{\parallel} of the wave be in the correct direction. The process utilizes the fact that transfer of energy between wave and particle is accompanied by a change in the particle's gyrocentre. Thus, diffusion in energy is coupled directionally with diffusion in space, and, with the correct choice of direction, a population of alphas diffusing in space under the influence of the wave will, on the average, transfer energy to the wave.

This possibility was discussed in a paper by Fisch and Rax [3], where they estimated the amount of energy transfer in the limiting case of infinite diffusion coefficient. In this paper we describe a finite-difference calculation used to solve the differential equation that models the process for finite diffusion coefficient. We simulate the limiting case (by taking a large value for the diffusion coefficient) to verify the analytic results of [3]. Small errors in the equations have been corrected. Then the energy transfer is calculated for a range of practical values and is found to

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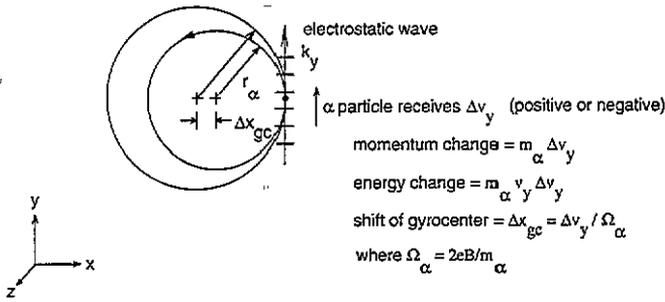


Figure 1. Schematic diagram of the coupling between the absorption of energy from the wave by an alpha particle and the shift of its gyrocentre.

be considerable. Finally, we modify the boundary conditions to introduce particle losses that enhance the energy transfer from particles to wave. The effect appears to be very promising.

2. The process

The process by which the energy and space diffusion are coupled has been described in [3]. It is summarized here. An example of the interaction is shown in figure 1, in which an alpha particle is encountered by a wave travelling the y -direction. As long as the velocity of the alpha, v_\perp , in the plane is greater than or equal to the wave phase velocity, ω/k_y , the two interact, and the alpha receives an increment of velocity Δv_y (positive or negative). As a result, the gyrocentre of the particle is shifted by an amount Δx_{gc} . The momentum change is $m_\alpha \Delta v_y$, and the energy change is $m_\alpha v_\perp \Delta v_y$. The gyrocentre shift is equal to $\Delta v_y / \Omega_\alpha$, where $\Omega_\alpha = 2eB/m_\alpha$. Therefore, $\Delta x_{gc} = \Delta E(k_y / m_\alpha \Omega_\alpha \omega)$, showing the coupling between energy transfer and spatial shift. The guiding-centre shift is inversely proportional to the wave phase velocity. We would choose a value like $\omega/k_y \sim 6 \times$ (ion thermal velocity), in order to avoid wave interaction with the thermal ions. For the case of total alpha energy transfer equal to the production energy, $E_{\alpha 0} = 3.5 \text{ MeV} = m_\alpha v_{\alpha 0}^2 / 2$, the guiding centre would move by $L = v_{\alpha 0}^2 k_y / 2 \omega \Omega_\alpha \sim 20 \text{ cm}$ in a $5 T$ field. Note that, for a given density gradient, more energy is extractable by making L long, that is, making the phase velocity low (so long as $L < a$, where a is the minor radius, which is generally satisfied anyway).

The poloidal wave numbers that are needed to accomplish the radial diffusion are much greater than are generally employed in current lower-hybrid heating experiments. In addition, the spectrum of waves must, of course, be launched in only one poloidal direction. For coupling both to the alpha particles and to fast electrons travelling in one direction, for enhanced current drive, the waves need to be launched also in one toroidal direction. The requirements on the wave (in the case of current drive) are that $k_\theta \gg k_\rho, k_\parallel$, where k_θ is the poloidal wavenumber, k_ρ is the radial wavenumber, and k_\parallel is the parallel wavenumber. Such wavenumbers obey the lower hybrid dispersion relation, where k_ρ/k_\parallel as large as $\sqrt{m_i/m_e}$ is allowed. The question of how to couple to these waves from the tokamak periphery is more difficult. There are solutions to the ray-tracing equations that do encircle the tokamak magnetic axis, with $k_\theta \gg k_\rho$ at least in the most interior points [4–6].

Achieving high k_{θ} , especially relative to k_{\parallel} , is not simple and may require toroidal effects on the ray tracing. Alternatively, the necessary conditions on the wave in the interior might be produced at high power by nonlinear wave interactions of waves that can be propagated easily from the periphery.

3. Analytic treatment

The differential equation describing the alpha density as a function of energy, position and time, under the influences of this 'quasilinear' diffusion, slowing down on electrons, and a source is [3]

$$\frac{\partial \rho}{\partial \tau} = \frac{\partial}{\partial \varepsilon} (\varepsilon \rho) + \left(\frac{\partial}{\partial \varepsilon} + \frac{\partial}{\partial X} \right) \frac{D}{\sqrt{\varepsilon - \varepsilon_w}} \left(\frac{\partial}{\partial \varepsilon} + \frac{\partial}{\partial X} \right) \rho + S(\varepsilon, X, \tau). \tag{1}$$

This equation is in terms of dimensionless quantities: $\varepsilon = v_{\perp}^2/v_{\alpha 0}^2$, $X = x_{gc}/L$, $\tau = 2vt$, where v is the slowing-down rate of alphas on electrons, $\varepsilon_w = (\omega/k_y v_{\alpha 0})^2$, the kinetic energy below which the alpha is not in resonance with the wave. ρ is the alpha density and S is the alpha source. The boundary conditions are determined from following: $D = 0$ for $\varepsilon < \varepsilon_w$, since there is no wave-alpha resonance there, and the wave exists only in a beam extending laterally from $X = 0$ to $X = A$ —outside that range $D = 0$.

The analysis of the behaviour of a population of alphas is, again, summarized from [3]. The process takes place in a space shown in figure 2. It will be convenient to introduce a transformation given by $u = \varepsilon + X$ and $v = \varepsilon - X$. The boundary conditions on the diffusion imply that $\partial \rho / \partial u = 0$ on the top and bottom boundaries. Since nothing can take particles across the right boundary, we can take $\rho = 0$ there. Only slowing down (and not diffusion) takes particles across the left, $\varepsilon = \varepsilon_w$, boundary, so one can write the flux at that boundary as $-\varepsilon \rho$; equivalently one can take the boundary condition $\partial \rho / \partial u = 0$. The diagram of the region of wave-particle interaction in this paper differs from that in [3], because here we draw the diagonal along which diffusion occurs at 45° to the right, a line of constant v . This is consistent with the form of the diffusion operator in equation (1). This difference

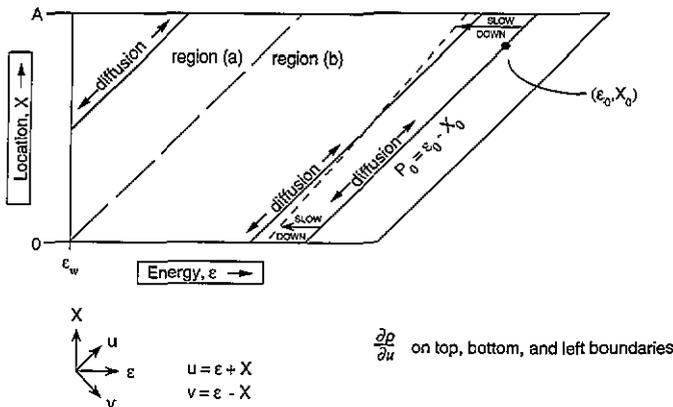


Figure 2. The effects of diffusion and slowing down on alpha-particle energy and location. The coordinates are dimensionless (see text). The source is at (ε_0, X_0) .

does not affect the conclusions, since it is merely equivalent to a replacement of X by $A - X$. One should think of $X = A$ being closer to the centre of the plasma, where the alpha-particle density would presumably be higher, so that the overall net energy transfer is from the particles to the wave. This is, of course, always possible by making the proper choice of k_θ .

Consider a short pulse of alphas produced at (ε_0, X_0) , i.e. a δ -function source. Consider the case in which $D \rightarrow \infty$. Initially the population diffuses along the 45° diagonal specified by $P_0 = \varepsilon_0 - X_0$, and the density immediately becomes uniform along that line. The energy exchanged, wave to alpha, is given simply by

$$W_0(X_0, \varepsilon_0) = \frac{1}{2}(\varepsilon_{\max} + \varepsilon_{\min}) - \varepsilon_0 \quad (2)$$

where the first term is the final average kinetic energy. Along the diagonal, $\varepsilon_{\max} = \varepsilon_0 - X_0 + A$, and ε_{\min} is given by

$$\varepsilon_{\min} = \begin{cases} P_0 & \text{if } P_0 \text{ is in region (b)} \\ \varepsilon_w & \text{if } P_0 \text{ is in region (a)} \end{cases} \quad (3)$$

(see figure 2). Substituting into equation (2), we obtain

$$W_0(X_0, \varepsilon_0) = \begin{cases} A/2 - X_0 & \text{if } P_0 \text{ is in region (b)} \\ (A - X_0)/2 - (\varepsilon_0 - \varepsilon_w)/2 & \text{if } P_0 \text{ is in region (a)}. \end{cases} \quad (4)$$

This instantaneous energy exchange can be positive or negative; for X_0 near A , it is negative.

The subsequent slowing down and diffusion are considered as a repeating alternation of slowing down for a short Δt , followed by diffusion for an equal Δt . Each slowing down takes the alpha population from a diagonal P to a left-shifted line tilted slightly toward the vertical. The diffusion then spreads the density uniformly along a band around the average 45° diagonal. As shown in [3], in region (b) the energy exchange vanishes.

In region (a) the energy exchange is no longer zero. The starting value of P depends on whether the alphas were produced in region (b) or (a). In the former case, the starting value is $P_{0a} = \varepsilon_w$; in the latter, $P_{0a} = P_0 = \varepsilon_0 - X_0$. Modifying the derivation in [3] for $X \rightarrow A - X$ and correcting minor errors† we have for the exchange subsequent to the initial exchange

$$W_L(P_{0a}, \tau) = \frac{\varepsilon_w}{1-s} \left(-\tau - \frac{1}{s} (e^{-\tau/2} - 1) + s(e^{\tau/2} - 1) \right) \quad (5)$$

where $s \equiv 2\varepsilon_w/(P_{0a} + A + \varepsilon_w)$. All the particles are gone by the time $\tau = \tau_{\max}$, where $\tau_{\max} = 2 \ln(1/s)$; see [3]. Therefore, if the rf power is on continuously, the ultimate value of W_L is

$$W_L(P_{0a}, \tau_{\max}) = \frac{\varepsilon_w}{1-s} (2 \ln(s) + 1/s - s). \quad (6)$$

† In equation (14) of [3], the sign in the denominator should be reversed, so that it would read

$$\frac{dN}{d\tau} = -\frac{\varepsilon_w}{P(\tau) - \varepsilon_w} N(\tau).$$

Nevertheless, equation (15) is correct. In equation (16) we have corrected the constant factors in front of the first and third terms in the brackets. As a result, each term has a factor of 4. A factor of 1/4 in front cancels these factors, giving finally equation (5) in this paper.

The total energy exchanged from wave to alpha is $W = W_0 + W_L$. In deriving equations (5) and (6) certain approximations were made, in addition to taking the limit $D \rightarrow \infty$. There is no limit in which these approximations are rigorous, but what we show (through numerical solution of equation (1) in the following sections) is that these analytic expressions for W_L are very nearly correct for $D \rightarrow \infty$.

Note that W_L is always positive. If X_0 is chosen so that W_0 is negative, meaning that the alphas give energy to the wave, then when they are in region (a), the wave 'gives back' energy to the alphas. For this reason, a scenario with only short pulses of rf appears advantageous.

4. Maximum energy transfer

The quantity ϵ_0 , the initial perpendicular energy, is given by the alpha production: the pitch angle determines an $\epsilon_0 \leq 1.0$. The distribution in ϵ_0 is uniform. To maximize the number of alphas diffusing to lower energy we need X_0 to be near the top of the region. In the practical situation k_θ would be chosen so that X_0 is toward the centre of the plasma with $X = 0$ toward the periphery. The quantity ϵ_w is the lowest perpendicular kinetic energy, relative to the total kinetic energy at production, of an alpha that is resonant with the wave. For putting the alpha energy into electrons, choose ϵ_w well above ion thermal energies, for example $\omega/k_\theta \sim 6v_T$, to avoid ion damping. This corresponds to ~ 500 keV and $\epsilon_w = 0.15$.

Using the analytic expression (6), we did a scan of the three parameters, ϵ_0 , ϵ_w and A , to locate the regions with the most energy exchange from alphas to wave. The energy exchange 'saturates' at around $A = 1$. Figure 3 shows the variation with ϵ_0 and ϵ_w for $A = 1$ and $X_0 = 0.95$. The initial energy exchanged is plotted, as well as the net taking into account the 'give back' during the alpha slowing down. Note that for a normal population of alphas, isotropic in velocity space, one would need to integrate over ϵ_0 to get the total energy transfer.

For the short-pulse rf scenario, clearly, the lower the value of ϵ_w , the more energy from alpha to wave; however, with continuous rf there is a broad maximum in the neighbourhood of $\epsilon_w \sim 0.2$. This is an additional reason for choosing ϵ_w in the vicinity of (but above) 0.15 mentioned above.

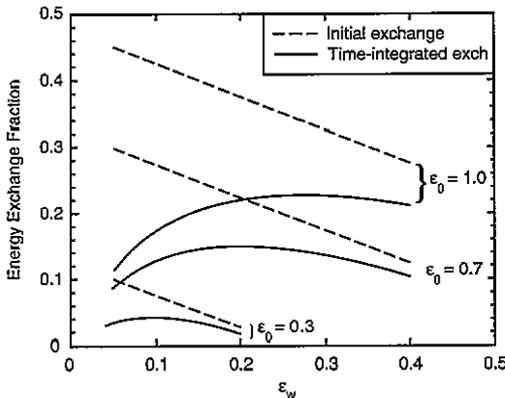


Figure 3. Energy exchange from alpha particles to wave as a function of the minimum resonance kinetic energy.

5. Comparison with numerical solution

The analytic study described above has been supplemented by a finite-difference solution of equation (1). This calculation allows us to model the effects of finite values of the diffusion coefficient, D , and altered boundary conditions intended to enhance energy exchange. This finite-difference solution can be used in the future to model other scenarios, such as applying the wave power in short pulses, also for the purpose of enhancing the fraction of the particle energy delivered to the wave.

We begin with the transformation $u = \epsilon + X$ and $v = \epsilon - X$. While it does not do good to transform the whole equation, it does simplify writing the diffusion term, since it implies $\partial/\partial\epsilon + \partial/\partial X = 2\partial/\partial u$. Figure 4 shows the grid that is used, along with the locations of diagonals and the points used in the central differences. For the diffusion term we choose the finite difference form to be space centred and use the Crank–Nicholson method [7]. For the slowing-down term, which has the form of a flow in the direction of decreasing ϵ , we use the ‘upwind’ difference [8]. This method is to take the difference between the point being worked on and the one from which the flow is coming; it gives stable solutions and has been widely used. The difference equation is

$$\begin{aligned} \rho_{j,k}^{i+1} = & \rho_{j,k}^i + \frac{\Delta\tau}{\Delta\epsilon} (\epsilon_{j+1}\rho_{j+1,k}^i - \epsilon_j\rho_{j,k}^i) \\ & + (1 - f_{imp}) \frac{4\Delta\tau}{(\Delta u)^2} [\kappa_{j+1/2}(\rho_{j+1,k+1}^i - \rho_{j,k}^i) - \kappa_{j-1/2}(\rho_{j,k}^i - \rho_{j-1,k-1}^i)] \\ & + f_{imp} \frac{4\Delta\tau}{(\Delta u)^2} [\kappa_{j+1/2}(\rho_{j+1,k+1}^{i+1} - \rho_{j,k}^{i+1}) - \kappa_{j-1/2}(\rho_{j,k}^{i+1} - \rho_{j-1,k-1}^{i+1})] + \Delta\tau S_{j,k}^i \end{aligned} \tag{7}$$

where $\kappa_j = D/\sqrt{\epsilon_j - \epsilon_w}$, and f_{imp} is normally set = 0.5. To make the method second-order in time, we take a half step, with $f_{imp} = 1.0$, to get half-step values for

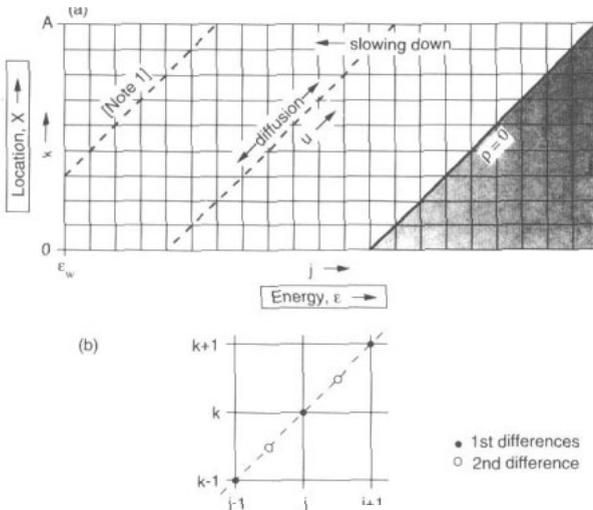


Figure 4. Grid for the finite-difference calculation of alpha particle energy and location. On the top, bottom and left boundaries $\partial\rho/\partial u = 0$, except in the case with leakage (see text). Note: iteration continues through the upper left corner.

the slowing-down term. The boundary conditions are $\rho = 0$ on the right (diagonal) and $\partial\rho/\partial u = 0$ on the left, top and bottom. First-order differences are used in implementing the boundary conditions. The sets of simultaneous equations generated by the implicit differences are solved by the method of Richtmyer and Morton [9]. We step through the diagonals, one by one; on each there is a system of equations for the ρ^{i+i} ; [9] gives a simple recursion method for solving the system subject to the boundary conditions.

The method and the computer program that implements it were tested by using a test function for the density, ρ . The function was chosen to satisfy the boundary conditions on all four boundaries. The function, ρ , was substituted into equation (1) to determine the corresponding source function, $S(\varepsilon, X, \tau)$. That source was used in the finite-difference program to calculate $\rho(\varepsilon, X, \tau)$ numerically. The results were compared with the assumed ρ to verify the method and to test the rate of convergence. The function chosen is the product of four polynomials. Substitution into the partial differential equation (PDE) leads to a large amount of manipulation, which, fortunately, is easily performed with the Macsyma symbolic manipulation system [10]. The Fortran code for the source function was also produced by Macsyma.

Figure 5 shows the results of the testing, monitored at two representative points. For the testing we used $D = 0.3$, $\varepsilon_w = +0.1$ and $A = 0.5$. The test function looks like a smooth hill, with the edges $=0$ at the boundaries and the peak at the centre of the space. The function was multiplied by an exponentially decaying factor, with time constant $\tau_{\text{decay}} = 0.0844$. The errors plotted are at time $\tau = 0.15$. The errors displayed are at points near the left-hand boundary, where the error is largest. The error varies more slowly than the first power of step size, apparently because of the factor $\sqrt{\varepsilon_j - \varepsilon_w}$ in the denominator. Nevertheless, the error is clearly converging on zero with decreasing step size, indicating the correctness of the method.

The goal of the calculation is to obtain the energy exchanged between fast alpha particles and the wave. To calculate this, once we have the alpha density, $\rho(\varepsilon, X, \tau)$, we multiply the PDE through by ε and integrate over all ε and X

$$\frac{\partial}{\partial \tau} \int \varepsilon \rho \, d\varepsilon \, dX = \int \varepsilon \frac{\partial}{\partial \varepsilon} (\varepsilon \rho) \, d\varepsilon \, dX + 4 \int \varepsilon \frac{\partial}{\partial u} \kappa \frac{\partial \rho}{\partial u} \, d\varepsilon \, dX + \int \varepsilon S \, d\varepsilon \, dX. \tag{8}$$

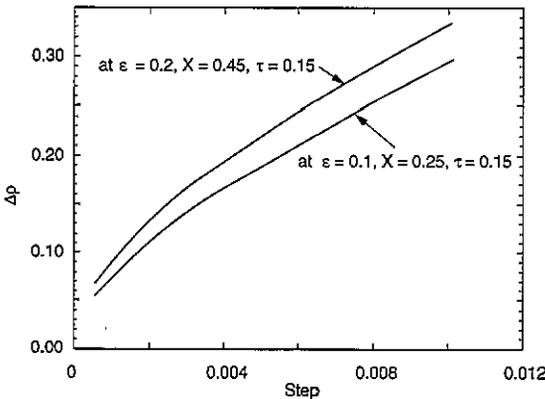


Figure 5. Difference between finite-difference solution and test function versus spatial (and energy) step size. The test function has an amplitude of 3.

This represents

$$\frac{\partial}{\partial \tau} \int (\alpha \text{ stored energy}) d\epsilon dX = \int (\text{power from electron drag}) d\epsilon dX \\ + \int (\text{power from the wave}) d\epsilon dX + \int (\alpha \text{ source power}) d\epsilon dX$$

where positive quantities represent power flowing *to* the alpha particles. The second term on the right is the power we are interested in. Under some conditions, in particular when the majority of alpha diffusion in space is accompanied by a lowering of alpha kinetic energy, the second term is negative, implying a net transfer of energy from alphas to the wave.

For comparison between the analytic and the finite-difference calculations we select two cases. Case I has $A = 1$, $\epsilon_w = 0.15$, $\epsilon_0 = 1.0$, $X_0 = 0.95$, which locates the source in region (a), and Case II is in region (b), with $A = 0.5$, $\epsilon_w = 0.10$, $\epsilon_0 = 1.0$, $X_0 = 0.4$. The latter case will demonstrate all the features of the predicted time evolution of the energy exchange, namely, a rapid initial exchange, followed by no exchange for the rest of the time in region (b), followed by a gradual return of a portion of the energy while in region (a).

To illustrate the behaviour of the energy exchange as a function of time, we first plot, in figure 6, alpha-to-wave energy exchange for Case II mentioned above (source in region (b)). As above in the analytic calculation, the source was taken to be a δ -function in ϵ , X and t . (Thus, the result here can be considered to be a Green's function for the energy exchange.) In this plot we have inverted the sign of the energy exchange so that alpha-to-wave is plotted as a positive quantity. Figure 6 is the result from the finite-difference calculation, integrated as described above. As expected from the analytic analysis, the energy exchange begins with an immediate transfer to the wave, after which the exchange during the rest of the time in region (b) is zero. When the alphas are in region (a), the wave 'pays back' some of the energy, but, by the time all of the alphas have slowed down below ϵ_w (in this case $\tau/\tau_s = 3$), there is still a net amount of energy that has been transferred to the wave. For this case the analytic analysis gives $W_0 = 15\%$ (using equation (4)) for the initial

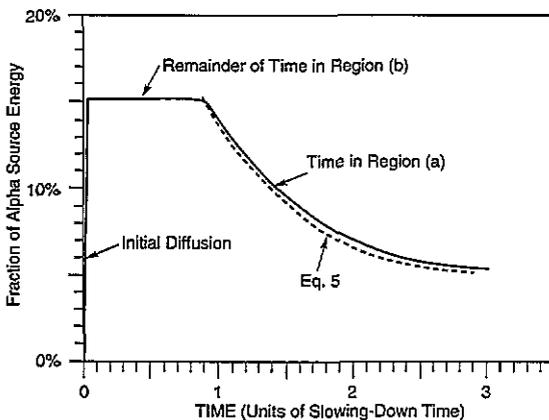


Figure 6. Time-integrated energy transfer from alpha particles to wave for Case II—source in region (b).

transfer from alpha particles to wave. Using equation (5) for the return of energy from wave to alpha particles gives the curve in figure 6 labelled 'Eq. 5'. The agreement is very good, verifying the analytic formula.

The convergence of the energies as a function of step size is very satisfactory. As the ϵ (and X) step was varied from 0.03 and 0.003, the energy unaccounted for at the end of the calculations went down from 12% to 1.3% and extrapolates to a value extremely close to zero for step size equal to zero. Also, the energy exchanged from alphas to wave extrapolates, as a function of step size, to the same value as the analytic calculation within $\sim 0.1\%$; the difference for the step size used for the results below (0.003) is 0.25%.

6. Optimal energy-transfer case

We now examine in detail Case I, namely, $A = 1$, $\epsilon_w = 0.15$, $\epsilon_0 = 1.0$, $X_0 = 0.95$. Although a slightly higher energy transfer to the wave could be obtained with $\epsilon_w \sim 0.3$, as may be seen in figure 3, the value chosen is more nearly suited to the average value of ϵ_0 and is more practical, since it would employ the lowest-frequency wave source consistent with the $6v_T$ requirement. In this case the source is in region (a), so there is no period of zero exchanged power. Results from the analytic formulae are as follows

$W_0 = 40.0\%$	(initial transfer)
$W_L = -19.5\%$	(later transfer)
$W = 20.5\%$	(net transfer).

The analytic formulae are for $D \rightarrow \infty$; the finite-difference calculation shows that the process saturates for $D = 10$ and above, where the analytic approximation, again, is numerically verified.

Figure 7 shows the energy transferred as a function of time for this case. When $D = 10$, as we have taken above, the behaviour is as expected for a source in region (a), that is, after the initial, immediate transfer, the 'give back' begins immediately. Interestingly, we find that, as the strength of D is reduced, its value has much more

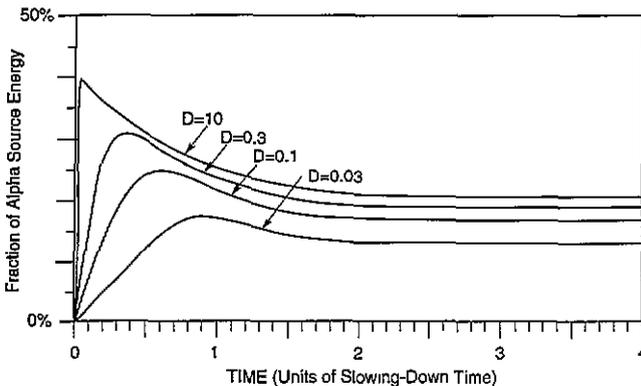


Figure 7. Time-integrated energy transfer from alpha particles to wave for Case I—source in region (a)—for a range of values of D , the diffusion coefficient.

effect on the early energy exchange than on the net amount. Likely values of D would probably be in the range $3 > D > 0.3$ [11], over which the net energy exchange varies only slightly. This has the fortunate implication that the strength of the wave can vary over a wide range while still effecting nearly optimal energy exchange.

7. Enhancing the energy transfer to the wave

Particles near the boundary $\varepsilon = \varepsilon_w$ can only take energy *from* the wave as they diffuse up along the diagonal; therefore, it would be advantageous to get rid of them. It would be possible to introduce perturbations into the plasma to cause particles to cross that boundary in excess of the collisional slowing down. To model the effect of such a particle loss, we have investigated changing the boundary condition on the left boundary, $\varepsilon = \varepsilon_w$, from $\partial\rho/\partial u = 0$ to $\partial\rho/\partial u + \lambda u = 0$, where λ is an adjustable 'leakage' parameter. The result is that a relatively small value of λ produces a large beneficial effect. This is shown in figure 8, where, as before, the energy exchanged as a function of time for a δ -function source is plotted. $D = 0.3$. The coefficient λ is in effect the fractional variation of the particle density per unit distance in u at the boundary. Starting from the net energy exchange of 19% for the case of no leakage, we find that $\lambda = 0.3\%$ increases the energy exchange to 26%, while $\lambda = 3\%$ takes it up to 51%. The strong value of $\lambda = 50\%$ yields 74% exchange. The possibility of enhancing the energy transfer to the wave, if such a means can be found, appears very promising.

8. Further applications of the program

Certainly, other scenarios can be modelled. One example would be using short pulses of rf, rather than continuous power, to derive the maximum benefit from the initial energy exchange (this would be an alternative to the lossy boundary).

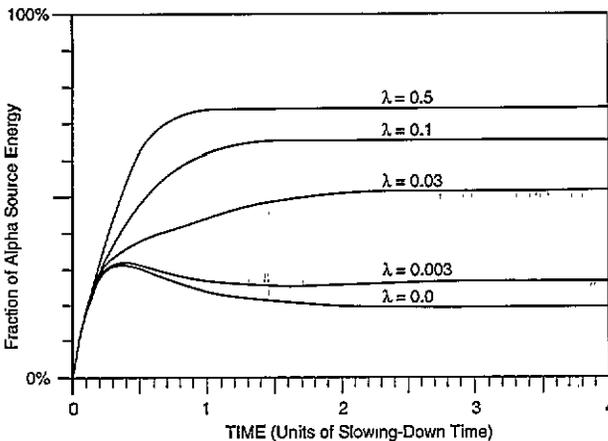


Figure 8. Time-integrated energy transfer from alpha particles to wave for Case I—source in region (a)—for a range of values of the leakage coefficient, λ . $D = 0.3$.

Another example would be allowing a general variation of D as a function of X , representing a more realistic localization of the wave in space.

9. Discussion

To look at the actual magnitude of the diffusion constant, D , and the power available from the alphas, consider ITER. In [2] the authors contemplate using 90 MW of lower hybrid power. This would give $D \sim 0.3$ to $D \sim 3$ [2, 11]. As shown above, this is in a range that produces practical energy exchange from the alpha population to the wave. The amount would be approximately the fusion power of 1000 MW, times a fraction of 20% that goes to the alphas, times a fraction of 40% that is transferred from the alphas to the wave. The result is 80 MW from alphas to wave, or just about the amount needed to keep the current drive going.

If the rate of normal collisional diffusion were to exceed the quasilinear diffusion considered in this paper, then the effect of the latter would be diluted. To make the comparison we can convert the dimensionless D used here to SI units. Recalling that our time is in units of $(2\nu)^{-1}$, and that length is in units of L , we have that the SI value is $D_{\text{SI}} = 2\nu L^2 D$. For the mean value $D = 1$, with $\nu^{-1} = 0.4$ s, and $L = 20$ cm, $D_{\text{SI}} = (0.2 \text{ m})^2 \times 1/0.2 \text{ s} = 0.2 \text{ m}^2 \text{ s}^{-1}$. One of the smallest values of D we have considered is $D \sim 0.1$, or $D_{\text{SI}} \sim 0.02 \text{ m}^2 \text{ s}^{-1}$. This is of the same order of magnitude as possible fast-alpha collisional diffusion [12]. In that case one would expect the energy exchange from alpha to wave to be diluted by the collisional diffusion. That puts a lower bound on the necessary wave intensity for an effective energy exchange.

10. Summary

We have provided numerical justification to the treatment of [3] for the $D \rightarrow \infty$ limit, we have shown numerically how that limit is approached for finite D , and we have numerically investigated the useful effect of leakage of particles out of the resonant region in optimizing the transfer of alpha-particle energy to waves.

Acknowledgments

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